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Financing Trade and the Price Level: Problems with the Description of Markets, Expectations, Money and Credit

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1. The Value and Drawbacks of Equilibrium Analysis

The general equilibrium system provides considerable insight into the functioning of the price system in a market economy in equilibrium but its very power and simplicity disguises several features which are critical in understanding an economy which uses money and credit.

Attempts to extend the competitive equilibrium analysis have, in different ways, been successful and have called attention to the proposition that the addition of more basic aspects of an economy, even at a high level of abstraction, point to the need to understand the monetary, financial and information processing aspects of the economy. In particular general equilibrium (hereafter GE) has been extended to general equilibrium with an infinite time horizon (GE∞). See for example: McKenzie (1986) and Koopmans (1977). In this extension the importance of expectations must be considered. A large literature on general equilibrium with incomplete markets (GEI) has grown up. See for example: Radner (1986). This literature shows the importance of uncertainty and the use of financial instruments to deal with uncertainty. A key insight linking biology and economics has been provided in the work on overlapping generations (OLG). See for example: Geanakoplos (1987). Here the importance of money, credit, population, inheritance and expectations is manifested in many of the models.

Another fruitful avenue of analysis has been temporary general equilibrium analysis (TGE), see for example: Hicks (1946), Grandmont (1982, 1983). Here much of the work has stressed the role of money and expectations.

It is, however, somewhat surprising, that in all of the work and various models noted above there has been hardly a mention of bankruptcy, default and reorganization. Yet a little reflection shows that in any system with the extension of credit, if the system is to be fully defined for positions which include all feasible states, not merely states in equilibrium, some will involve situations in which a debt cannot be repaid, and thus bankruptcy and reorganization becomes a logical necessity. There is a small literature concerning default in a closed economy by Shubik and Wilson (1977), Dubey and Shubik (1979), Dubey, Geanakoplos and Shubik (1988), Zame (1993) and Karatzas, Shubik and Sudderth (1994). But to date this has apparently had little impact on the thinking of those working within the GE framework.

2. Strategic Market Game Analysis

An alternative, but highly related approach to the variants of GE analysis noted above is to study the price system by means of Strategic Market Games (SMG). This approach is based on my belief that GE theory in all of its variants can benefit from a basic shift in its outlook from an emphasis on equilibrium to an emphasis on process. The economy is dynamic, yet except, to some extent, in GE∞ and TGE the dynamics hardly appears.

The approach towards dynamics can be broken into two parts. The first involves the full description of the economy as a game of strategy whose set of outcomes is defined and described independently from any specification of equilibrium or discussion of the behavior of the individuals.

Given a fully defined game, the set of rules of the game provides an elementary description of the economic institutions of the society. With the game as given we may then wish to impose a particular form of solution which we may believe provides insights into interest and behavior. Thus a natural question to consider is, is there a relationship between the noncooperative equilibria (NEs) of a strategic market game and the related GE model based on the same elementary economic data? When the economy involves many small agents with essentially no individual market power, the answer to this question is: yes, often. The techniques for solving strategic market games with many small agents (technically: a continuum of non-atomic agents) are such that some might feel that a distinction is being made between GE and SMG that hardly shows a difference. This is not so, using the test of a model that is a completely defined playable game. The SMG provides a complete process description regardless of equilibrium.

Many years ago, I coined the phrase *Mathematical Institutional Economics* (Shubik, 1959). This was designed to stress that at an abstract level the rules of the game describe the elementary economic institutions and instruments of a society. They are the carriers of process.

An immediate concern of the theorist striving after a high level of generality is that modeling fine structure such as price formation or check clearing involves a level of detail that is overly special and not suited to use to obtain general conclusions of interest. In contrast with this view, the approach adopted here is: (1) The requirements of consistency and completeness call for the specification of all of the rules of the game. (2) There may only be a few mechanisms which can be defined as "minimal institutions," for example, in a mass market with simultaneous moves and little information the Cournot and Bertrand–Edgeworth models may provide the simplest or minimal mechanisms for price formation. (3) After one has obtained a well-defined model, then it is of interest to ask is it possible to produce axioms which describe a large class of mechanisms which might display the properties desired? This approach is illustrated by Dubey, Mas–Colell and Shubik (1980) and Dubey, Sahi and Shubik (1993).

A test which I believe will become of increasing importance in the development of verifiable economic theory is the test of a playable game. Not only should an economic model be a full process model but one should be able to use it, or a near approximation of it, as an experimental game which can be understood by players who do not need a Ph.D. in mathematical economics in order to be able to play.

3. A Study of the Stationary State

In an attempt to study the many aspects of the money supply it is possible to divide up several of the difficult features and to study them separately. In particular the key division is between economies which are in a full stationary state and hence require the same amount of money and credit each period, and economies which require a variable money supply. Several further subdivisions are in order. The stationary state economies can be divided into those with everliving agents and OLG economies.

The economies requiring a variable money supply can be further characterized into those with stochastic or nonstochastic future monetary needs.

In this paper we limit our concern to stationary state economies with everliving agents. This enables us to avoid considering several key problems involving banking and control over the varying of the money supply. We concentrate on understanding the roles of money, credit, credit limitations, clearinghouses, bankruptcy, markets, inside and outside banks in financing even a stationary state. Variations of a simple model of an exchange economy are used to illustrate the role of the financial structure.

In particular the points to be illustrated are:

(1) The relationship or lack of relationship between the "cash-in-advance" and one period time-lag in payments models.

(2) Problems in forecasting and the formation of expectations.

(3) The importance of bankruptcy and reorganization.

(4) The critical role of bounds on credit in bounding price.

(5) The critical role of bounds on credit in limiting expectations.

(6) The upper and lower bounds on the price system.

4. Financing Trade in a Stationary Economy

4.1. A Simple Exchange Economy with the CE Solution

In this and subsequent sections we consider a simple exchange economy examined for its competitive equilibrium solution, then the same economy is remodeled as a strategic market game in several different ways reflecting different financial and credit arrangements.

We consider an economy with two types of trader, all of whom have the same utility function defined over the infinite horizon and of the form

$$U_i = \sum_{t=0}^{\infty} 2\beta^t\{\sqrt{x_t^i} + \sqrt{y_t^i}\}.$$

Where x_t^i, y_t^i are the consumption levels by i during t of the first and second perishable commodities available each period.

There are n traders of each type where traders of type 1 obtain (2, 0) each period, i.e. two units of the first perishable and traders of type 2 obtain (0, 2). It is easy to observe that the trivial CE solution has prices $p_{1t} = p_{2t} = p$ where $0 < p < \infty$ and $x_t^i = y_t^i = 1$ hence:

$$U_i = \sum_{t=1}^{\infty} 2\beta^t\{\sqrt{1} + \sqrt{1}\} = \frac{4}{1-\beta}.$$

4.2. A Strategic Market Game with Clearinghouse Credit

The competitive equilibrium (CE) of the general equilibrium model of exchange (GE∞) was so simple that it could be solved at a glance. But in this solution nothing is learned about economic process or financial needs. Here we consider a playable game which fully describes process.

In order to stress the playable game aspects we first model it as a finite player, finite horizon game, then can easily adjust to the infinite horizon with a continuum of agents.

As before there are n agents of each type. The game lasts for T periods. The utility function of each individual is of the form:

$$U_i = \sum_{t=0}^{T} 2\beta^t\{\sqrt{x_t} + \sqrt{y_t}\} + \sum_{t=0}^{T} \beta^{t+1}\mu \ \min[\text{Debt}_t,\ 0].$$

The reason for this form is explained below as we construct the process model:

Assumption 1. *The sell–all condition*

In a modern economy few small agents are self-sufficient or in a position to hold back from the market whatever they have to sell. For simplicity, in this model, we assume that all goods are offered to the market. This extreme assumption has several desirable features. It provides an extreme upper bound for the amount of money needed in an economy, as all assets are monetized every period. It lies at the other extreme from the GE model where all individuals have control and make decisions concerning the sale of an arbitrary number of goods. For the most part most individuals sell few goods and services relative to the number they buy.

In Section 5.3 when land, a producer asset is introduced we modify sell–all.

Assumption 2. *Price formation*

In Assumption 3 the form of money and credit is discussed. Let the bid in period t by a trader of type i for good j be b^i_{jt} where $i = 1, 2$, $j = 1, 2$ where b (which may be some form of credit or fiat) is denominated in terms of a numeraire.

The price of good j at period t is given by:

$$p_{jt} = \frac{\sum_{k=1}^{n} b^k_{jt} + \sum_{t=1}^{n} b^i_{2t}}{2n}.$$

Assumption 3. *Money or credit*

We assume that there is a numeraire and, in particular, debts and the default penalty are denominated in terms of the numeraire. We further assume here that there is a clearinghouse such that each individual has a clearinghouse credit line denominated in units of the numeraire, say, for example, dollars. Individuals issue their own IOU notes in dollar amounts and at the end of each period the clearinghouse matches all debts with the credits obtained from the sale of owned assets in the market. Thus for example the net debt of an individual i of type j at the end of period t is:

$$\text{Debt}^i_t = -b^i_{1t} - b^i_{2t} + 2p_{jt}.$$

In this first model we assume that the only type of "money" in this system is clearinghouse credits. This enables each individual to create his own paper. Two problems with this arrangement must be solved before we have a playable game. They involve whether the credit line granted to each individual should be bounded, what constitutes a default and what happens in a default. We consider bounds on credit first.

ASSUMPTION 4. *The line of credit is bounded*

Can individuals issue IOU notes of unlimited size? Although one can envision a game in which there are no constraints whatsoever on the issue of IOU notes and the acceptance of an individual's notes will depend on his reputation for default. In designing a playable game, a reasonable simplification is to have the clearinghouse specify an upper bound on the amount of IOU notes any individual can issue. Suppose that this amount is $B^{ik} > 0$ in each period. Thus in a single period game the full strategy set of an individual i of type k is $(b_{11}^{ik}, b_{21}^{ik})$ where

$$b_{j1}^{i} \geq 0 \text{ and } 0 \leq b_{11}^{ik} + b_{21}^{ik} \leq B^{ik}.$$

ASSUMPTION 5. *The bounding of credit, evaluation and expectation*

A reasonable assumption concerning the selection of bounds in the granting of credit is the estimation of an individual's ability to pay. In this game this depends on the estimated prices $\widehat{p}_{it}$ and $\widehat{p}_{2t}$ as type 1 individuals will earn $2p_{1t}$ and type 2 will earn $2p_{2t}$. Thus B^{ik} is a function of $\widehat{p}_{1t}\, \widehat{p}_{2t}$ but these, in turn, are functions of the history of the game.

Given the first bounds B^{ik}, however they were obtained, they place a bound on expected price

$$p^{jt} \leq \frac{nB^1 + nB^2}{2n} = \frac{B^1 + B^2}{2}.$$

ASSUMPTION 6. *Default, bankruptcy and reorganization*

If this game is to be actually playable by a group of individuals unless all play the same strategy some players might find themselves in debt at the end of a period. The rules of the game require that we specify what happens in this instance.

We offer a simple set of rules. If an individual has a positive balance his debt is discharged and his credit is noted. If an individual has a net debt it is discharged by the clearinghouse collecting whatever it can, the debtor is punished by a factor of $\mu\times$ net debt where μ is selected to be high enough to discourage strategic bankruptcy at a possible equilibrium.[1] The aggregate of the losses will offset the credits which are then written down to zero.

If bids are all bounded then prices will be the sum of all money bid for a good divided by the quantity for sale.

ASSUMPTION 7. *Information and strategy sets*

In multistage games it is well known that strategies proliferate with the increase in information. In the actual playing of a multistage game it is reasonable to assume that individuals tend to make decisions about what to bid each period based on their knowledge of previous prices and possibly even detailed information on the previous actions of others.

The equilibrium points of a game are often sensitive to the information conditions. The extremes in information are when the individual has perfect information. This would require sequential moves so that all information sets are one element sets; or when an individual picks up no new information whatsoever. This game stacks the conditions against complex strategies and amounts to the extreme situation where a player has no recall of the past. It nevertheless can be played even as an experimental game for T periods by defining a strategy for an individual i of type j to be of the form

$$(b_{11}^{ij},\, b_{21}^{ij},\, b_{12}^{ij},\, b_{22}^{ij},\, \ldots,\, b_{1T}^{ij},\, b_{2T}^{ij})$$

where

$$b_{1t}^{ij} \geq 0,\ b_{2t}^{ij} \geq 0,\ b_{1t}^{ij} + b_{2t}^{ij} \leq B^j.$$

This is adequate provided that the credit line is not changed from period to period. If it were the strategies could be defined in terms of percentages of the credit line to be bid, without knowing what the actual line is

$$b_{1t}^{ij} = \xi B_t^j,\ b_{2t}^{ij} = (1-\xi)B_t^j \text{ where } 1 \geq \xi \geq 0.$$

We are now in a position to analyze this simple game for 1, T and ∞ periods and for various combinations of $(B,\, \mu)$ the credit limit and the default penalty.

[1] When there is no exogenous uncertainty bankruptcy will be strategically motivated if the penalty is not high enough, unless it is caused by incompetence. When exogenous uncertainty is present strategic bankruptcy and legitimate risk taking are confounded. Phenomena such as adverse selection and moral hazard cannot be avoided.

Starting with the one-period game, an individual of type i will try to maximize

$$2\{\sqrt{b^i_{11}/p_1} + \sqrt{b^i_{21}/p_2}\} + \mu \ \min[(2p_i - b^i_{11} - b^i_{21}),\ 0].$$

It can be easily checked that for this credit money model we obtain the same distribution of goods at the NE as at the CE for any n, however for (B, μ) where $\mu = \frac{B}{2}(\frac{n-1}{n})$ the price system is unique and $p = p_1 = p_2 = \frac{B}{4}(\frac{n-1}{n})$ with $b^i_{11} = b^i_{21} = \frac{B}{2}(\frac{n-1}{n})$.

Suppose $\frac{B}{2} > \varepsilon > 0$ and $\mu = \frac{\varepsilon}{2}(\frac{n-1}{n})$ then the price level may be anywhere between $(\frac{n-1}{n})\frac{\varepsilon}{4} \leq p \leq \frac{B}{4}$.

4.3. Strategies, Expectations and the T Period and Infinite Horizon Games

The central aspects of money and credit are information processing and trust. Before considering the many period games we note the information structure for the one period game. Figure 1 shows traders bidding simultaneously in two markets for goods. The debits and credits are flowed through from the markets to a clearinghouse where, after clearing, final settlement is called for directly by the clearinghouse to the traders.

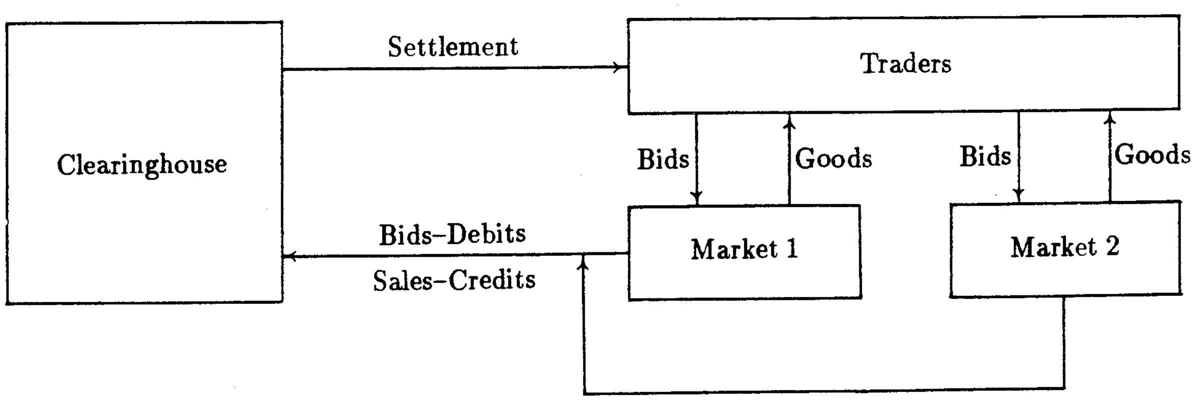

Figure 1

This game is administratively easy to play for one or for several periods. But the contrast between how individuals play and the game theoretic or general equilibrium description of equilibrium does not provide a satisfactory picture of the means for arriving at an equilibrium. It is as though the story one can offer to justify long-run equilibrium and rational expectations is that each individual is split into two — a long-run forecaster who estimates the aggregate behavior of others into the distant future, without telling us how he does the estimates and a naive but high IQ corporate planner who accepts the estimates and optimizes using them as a basis. This optimization, for the infinite horizon, may easily involve solving a dynamic program of some complexity, without even the benefit of justifying the predictions by backward induction. Instead we may invoke "rational expectations" as a means for justifying a solution which shows nothing more than the logical possibility for the existence of a stationary equilibrium and provides neither a prescription nor a description of how such a state is achieved.

The problem of expectations and their formation is key to understanding of multistage economic models. Here, it is suggested that in the construction of process models with the mechanisms for the creation and distribution of money and credit instruments made clear, at least new bounds on expectations can be calculated.

A Strategic Market Game with Cash Only

We modify the model in Section 4.2 which had clearinghouse credit only to the same economy with fiat money only.

ASSUMPTION 1. *Sell–all*

Although in the "real world" (whatever that is) sell–all is clearly not quite true, in an economy not breaking down with hyperinflation or regressing into barter it is probably a better approximation of economic life than the deceptive generality of the GE models. In this model, as all goods must be sold, even for a finite T the backward induction starting at period T does not wipe out trade or the use of money even though it may be worthless at period $T+1$.

ASSUMPTION 2. *Price–formation*

This is as in Section 4.2.

ASSUMPTION 3. *Fiat money*

We assume no credit whatsoever, hence no need for default rules. In contrast with Figure 1, Figure 2 shows no need for a clearinghouse and "cash–only" purchases remove the need for much of the record keeping. The individual markets need to record ownership claims in order to pay out incomes obtained from selling the goods.

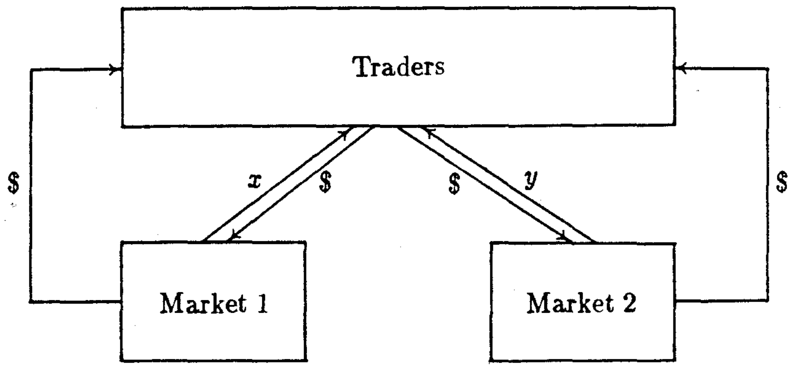

Figure 2

ASSUMPTION 4. *The money supply is finite and given*

In this simple example there is no economic need to vary the money supply. The equivalent model to Section 4.2 would have instead of the line of credit, the individuals have cash. In particular the endowment of type 1 is $(2, 0, M/2)$ and type 2 is $(0, 2, M/2)$ where M is the total amount of money the system.

It is easy to observe that if M_t^{i1} = the amount of money held by a trader i of type 1 at the start of time t, then:

$$M_0^{i1} = M/2 \text{ and } M_{t+1}^{i1} = M_t^{i1} = M_t^{i1} - b_{1,t}^{i1} - b_{2,t}^{i1} + p_{1t}$$

and similarly for type 2.

If we restrict our analysis to the low information game then a strategy for a trader i of type j in the infinite horizon game is of the form

$$(\widetilde{b}_1^{ij}, \widetilde{b}_2^{ij}) = (b_{10}^{ij}, b_{20}^{ij}; b_{11}^{ij}, b_{21}^{ij}; \ldots, b_{1,t}^{ij}, b_{2,t}^{ij}, \ldots)$$

where after period $t = 0$ each $b_{1t}^{ij} = \xi M_t^{ij}$ and $b_{2t}^{ij} = (1-\xi)M_t^{ij}$ where $0 \leq \xi \leq 1$.

The unique noncooperative equilibrium point (NE) is given by each trader bidding:

$$\left(\frac{M}{4}, \frac{M}{4}; \frac{M}{4}, \frac{M}{4}; \ldots\right)$$

and spot prices become

$$p_{1t} = p_{2t} = M/2.$$

An important distinction between the clearinghouse credit model of Section 4.2 and the fiat money model is that in the former the price level was not uniquely determined but was in a range determined by μ and B, but in the latter the price level is fixed by the money supply. There can be no equilibrium with hoarding because as money is an asset not a debt any individual can improve in a stationary state by spending any money hoarded. This is not true if there were cyclical needs — then all might wish to hoard.

5. Inside or Outside Banking

5.1. A Clearinghouse or Outside Bank?

Historically central banks, managing the money supply have been a relatively recent development from the late seventeenth century (see: de Kock, 1974, Goodhart, 1988). Early money was a commodity coined with the permission of the authorities. Debt, which historically came into existence prior to coinage was contracted between individuals and could be denominated in a weight of a precious metal or in other assets.

As the "outside" or central bank is a relatively new invention in the world of finance. It serves many purposes of government relating to the specialized functions and powers of government. A key feature, nevertheless is its role as the controller of the fiat money supply.[2] But this function has come into being because, unlike debt which is a contract, money was a physical asset. But fiat money is an abstract or "virtual" asset and if it is to function as a paper gold, the outside bank must provide the controls.

[2] Institutionally some distinctions may be made in the control roles of a treasury and a central bank, but at this level we treat them as one.

We can construct a model of an economy with an outside bank used for financing a stationary economy which gives us a playable game which at a high level of abstraction is identical with the model of a clearinghouse in Section 4.2. Instead of treating fiat money as an individually owned asset we could treat it as a debt contract between the holder of the fiat and the state who is regarded as the initial lender. With this model, at the end of any finite game the fiat must be returned to extinguish the debt. But if there is no inside borrowing or lending the trade books must balance each period and the whole structure is operationally the same as a clearinghouse.

If the needs of trade and innovation call for a variation in the money supply possibly a central clearinghouse could serve this function by changing the size of credit lines. But this calls for the power to set financial policy in the issue of credit. This function is ignored in economic theory and is possibly central in understanding the differentiation in the money supply function of the commercial and the central bank.

5.2. A Clearinghouse and Private Banks in an Exchange Economy

In this model we contrast the roles of the granting of credit with those of lending money. In a completely stationary economy, of the simple variety considered here the only need for money or credit is to finance transactions or, in essence, the float. Individuals can pay cash or use some form of credit which is then cleared. There is no need to borrow for investment or intertemporal trade or innovation.

We construct a model where there is both a clearinghouse and a new set of players who have as their only asset, money.

We extend the model in Section 4.2 by introducing a new set of players who have exactly the same utility functions as the others, but they possess only money. In particular the endowments of the three types of agents are as follows:

Type 1 $(2, 0; 2, 0), (2, 0; 2, 0), \ldots$

Type 2 $(0, 2; 2, 0), (0, 2; 2, 0), \ldots$

Type 3 $(0, 0; 0, 4), (0, 0; 0, 0), \ldots$

where the four entries are the new endowments of the first good, the second good, the credit line of the clearinghouse and the new supply of fiat money. Each period new supplies of the goods enter and the credit line is extended, but fiat money is introduced only once at the start.

In the first period traders of the third type could buy half of all goods available. The first and second types use their credit lines to the full, bidding 1 each in both markets, the third bids 2 in each market. Thus prices are 2, 2 and the final consumption in the first period is $(\sqrt{1/2},\ \sqrt{1/2})$ for types 1 and 2 and $(\sqrt{1},\ \sqrt{1})$ for type 3. In subsequent periods the first two types obtain $(\sqrt{1},\ \sqrt{1})$ and the third obtains nothing. This however is not an equilibrium.

If lending is permitted the infinite horizon NE solution has $p_{1t} = p_{2t} = 2$ and the stationary state payoffs are $(\sqrt{3/8},\ \sqrt{3/8})$, $(\sqrt{3/8},\ \sqrt{3/8})$, $(\sqrt{1/4},\ \sqrt{1/4})$. The players of the third type lend 2 (or 1 each) to players of type 1 and 2 at a rate of interest of $\rho = 100\%$. Thus types 1 and 2 spend 3 each, type 3 spends 2 and lends 2. Types 1 and 2 earn 4 each and pay 2 each to type 3. This process repeats indefinitely.

At a slightly higher level of generality we can consider an initial distribution of $(2,\ 0,\ c,\ 0)$, $(0,\ 2,\ c,\ 0)$, $(0,\ 0,\ 0,\ 0)$ for $0 \le c < \infty$. We observe that when $c = 0$ we have the economy in which the bankers can take half of the product. As c becomes larger the power of the bankers diminishes approaching zero. In essence, the stationary state proportions, when $\beta = 1/2$ will be $\left(\frac{c+1}{2c+4},\ \frac{c+1}{2c+4},\ \frac{2}{2c+4}\right)$. This approaches the "cashless society".

5.3. The Introduction of a Production Asset: Two Solutions

We take a small step in the direction of a richer model by introducing a productive asset. Instead of having the "manna" or the consumer perishable enter exogenously each period we consider a producer durable "land" which produces the consumable.

Many productive assets trade infrequently in relation to their length of existence. Thus, although the "sell–all" assumption is reasonable for consumer perishables it is not reasonable for land. In the model here, we assume that individuals who own land have the choice to sell or hold inventory, but they must offer the perishable to market. Reconsidering the model of Section 5.2, we assume an initial distribution of assets of:

Type 1 (1, 0, 2, 0; 2, 0)

Type 2 (1, 1, 0, 2; 2, 0)

Type 3 (0, 0, 0, 0; 0, 4)

where in the 6 dimensional array, entry 1 is the unit of land producing two units of the first perishable, 2 is the unit of land producing two units of the the second perishable, the 3 and 4 entries show the amounts of the two perishables produced each period. Production takes some short time Δt and land is sold with its current product. Land does not depreciate. Entry 5 is the clearinghouse credit and entry 6 the amount of fiat held.

Without formally writing out the details of the strategy sets, it is fairly straightforward to observe that there are, for the game with a continuum of agents, two solutions.

SOLUTION 1 *Lending only*

No land is put up for sale, 2 units are lent by the bankers (type 3) and two are spent. Interest is endogenously determined to be 100% or $\rho = 1$. Final consumption is $(3/8, 3/8), (3/8, 3/8)$ and $(1/4, 1/4)$. If $\mu = \frac{1}{2\sqrt{3/8}} = \sqrt{2/3}$ prices will be uniquely determined at $p_1 = p_2 = 2$.

SOLUTION 2 *The sale of land*

Each of the first two types offer 1/4 of a unit of land for sale. The third type bids 1 for each thus the price of land is $p_1 = p_2 = 4$ and of food $p_3 = p_4 = 2$. Consumption is as above and nothing is lent.

There are also solutions with between 0 and 1/4 of land sold and between 2 and 0 borrowed.

DISCUSSION

The presence of a nonproducible infinite horizon capital stock provides an alternative to borrowing and offers a "backing" for the fiat in the sense that the capital stock provides the consumption stream of goods and those who hold fiat can spend it to purchase part of the real assets equal in present value to the stream of consumption could obtain by lending and spending the earnings on consumption in an optimal manner.

6. Concluding Remarks

The discipline of requiring that full physical models be constructed of an economy capable of carrying process, forces us to pay attention to central items in the studying of the role of money, markets and finance which are often overlooked or dismissed as being institutional and of little general theoretical interest.

In particular different ways of looking at or interpreting commonly recognized phenomena emerge. In this section the six items noted in Section 3 are dealt with.

(1) Cash-in-advance

In macroeconomics, much has been made of the "cash-in-advance" constraint suggested by Clower and utilized extensively by Lucas. This constraint is almost always treated as a one-period time lag, but from the game theoretic point of view a similar constraint emerges as a *logical requirement* in defining the strategy sets of individual players in such a manner that their actions form prices rather than take prices as given. The phenomenon which must be accounted for is *the financing of the float* which is created by the many independent actions which form price and permits efficient settlement. There is no logical need for income to be lagged a full period in a strategic market game. In many economic and financial activities the natural lag may be a day or several days or (previously) even weeks required for settlement.

Payment in gold, fiat money, bank credit or plastic is highly a function of the customs, legal structure, level of trust and information processing and evaluation capabilities of the economy involved.[3]

(2) Forecasting and expectations

As soon as credit arrangements are considered the lenders must estimate the ability of the borrowers to repay, but this requires that they forecast prices and expected incomes. How they

[3] The full consideration of several basic aspects in modeling the float is not given here, but it is noted that a three parameter modification of the cash flow conditions arising in a strategic market game is probably adequate. The three parameters are as follows: (1) The percentage of all goods which must pass through the markets each period is given by $0 \leq \theta \leq 1$. (2) The percentage of all goods for sale whose sale can be settled through clearing without using cash or bank credit each period is given by $0 \leq \eta \leq 1$. (3) Of those transactions requiring cash or bank money the minimum percentage of cash required is given by $0 \leq \omega \leq 1$.

do so is one of the critical open problems in finance and economics, but the need to do so is a matter of necessity in well-defining the process.

(3) Bankruptcy and reorganization

Not only are the bankruptcy and reorganization rules also logical requirements, but a little reflection indicates that in an economy with incomplete markets, even if equilibrium exists, the equilibrium state may involve a number of individuals going bankrupt each period. The analogy with inventory theory should help to make this point. In general an optimal policy will risk some out-of-stock level for inventory; but here the inventory is money. The acceptable equilibrium level of bankruptcy in a society is a public good and is closely related to the overall willingness of society to absorb the consequences of risktaking. The bankruptcy level is closely related to the generation of innovation in an economy, in essence it controls the economic mutation rate.

(4) The critical role of bounds on credit in bounding price

I suggest that at any point in time credit agencies have an upper bound on their beliefs concerning short term expected prices, incomes and volume of trade. These upper bounds are used in placing upper bounds on the permitted borrowing of any would be borrower. No one is in a position to borrow twice the World National Product, no matter what he promises. But this clashes with the usual comfortable partial equilibrium assumptions concerning unrestricted lending in competitive markets. As yet we have no generally accepted and clearly worked out general equilibrium or disequilibrium models where the amounts of money and various forms of credit play an explicit role.

If both credit and money are bounded, then so are prices and prices at any point of time are no longer homogeneous of order zero in an unrestricted manner as an upper bound has been imposed.

(5) The limits on expectations

Even in a hyperinflation, if there is a physical limit to the speed with which new money can be printed or credit instruments issued, there are some bounds imposed on expectations. We obtain the usual sort of circular relationship postulated for both rational expectations and the existence of a noncooperative equilibrium. If all believe in an appropriate manner then the predictions of all will be self-fulfilling. But the beliefs themselves, to be consistent, must reflect the physical and legal constraints imposed on the credit mechanism.

A key characterizing feature of hyperinflations and bubbles is where the credit granting agencies start to change the rules, increasing the borrowing bounds. But even so, at any point in time these bounds are finite and credit is *never* unbounded.

(6) The upper and lower bounds on the price system

Price in a general equilibrium system is defined on the open interval $(0, \infty)$. When this system is remodeled as a strategic market game with bankruptcy and reorganization rules specified and monetary and credit instruments bounded in quantity then the equilibrium price system is defined on a closed interval $[p_*, p^*]$ where the lower bound is determined by the bankruptcy conditions and the upper bound by the limits on the supply of money and credit.[4]

[4] There is clearly an arbitrary choice in the normalization of what is meant by a unit of money, but once this is fixed and the bankruptcy conditions and penalties are related to the monetary unit the system is determined.

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